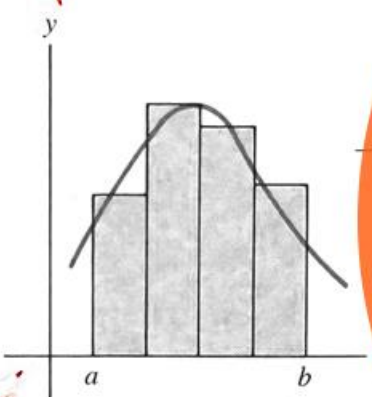
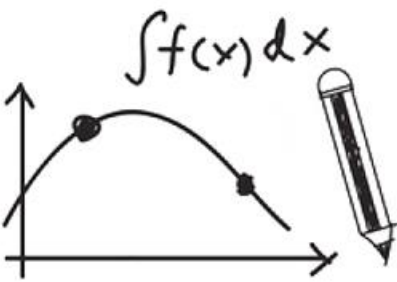


Calculus(I)

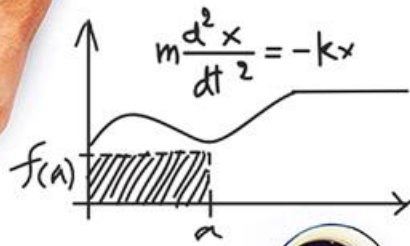
$$x^2 - 3x - 4 = 0$$

$$4x^2 - 3x - 1 = 0$$



$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$

$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \frac{dT}{dt} - (c_1)(T - T)$$

$$\frac{df(x)}{dx}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$cx + h, f(x + \tau)$$



2.7 Implicit Differentiation

Lecturer: Xue Deng

Problem Introduction

Def: $F(x, y) = 0$ a function, the corresponding function $y = y(x)$ satisfying $F(x, y(x)) \equiv 0$ is called implicit function.



Let $\sin y + xe^y = 0$, we cannot solve for y in terms of x ,

In theory, $y = y(x)$ satisfying $\sin y(x) + xe^{y(x)} \equiv 0$,

How to find the derivation of y ?

Implicit Differentiation

Derivation Rule of Implicit Function

Method: Derivation about x on both sides of the equation.


Tips: the variable y is a function of x .

The function of y is a composite function of x .

Use the composite chain rule for y .

Example 1

Find dy/dx if $x^2 + 5y^3 = x + 9$.

 $\frac{d}{dx}(x^2 + 5y^3) = \frac{d}{dx}(x + 9)$ $y = y(x)$

$2x + 15y^2 \frac{dy}{dx} = 1$



$\frac{dy}{dx} = \frac{1 - 2x}{15y^2}$

Example 2

$$\sin y + xe^y = 0, \text{ find } f'(x).$$

$$\pencil \cos y \cdot y'_x + 1 \cdot e^y + xe^y \cdot y'_x = 0 \quad y = y(x)$$

$$\Rightarrow y'_x = \frac{-e^y}{\cos y + xe^y}.$$

Example 3

Find the equation of the tangent line to the curve

$$x^3 + y^3 = 3xy \text{ at the point } \left(\frac{3}{2}, \frac{3}{2}\right).$$



$$3x^2 + 3y^2 \cdot y' = 3y + 3xy'$$

$$\therefore y' \Big|_{\left(\frac{3}{2}, \frac{3}{2}\right)} = \frac{y - x^2}{y^2 - x} \Big|_{\left(\frac{3}{2}, \frac{3}{2}\right)} = -1$$

The tangent line:

$$y - \frac{3}{2} = -\left(x - \frac{3}{2}\right) \implies x + y - 3 = 0.$$

Example 4

Find dy/dx if $4x^2y - 3y = x^3 - 1$.



We can solve the given equation explicitly for y as follows:

$$y(4x^2 - 3) = x^3 - 1 \qquad y = \frac{x^3 - 1}{4x^2 - 3}$$

Thus,

$$\begin{aligned} \frac{dy}{dx} &= \frac{(4x^2 - 3)(3x^2) - (x^3 - 1)(8x)}{(4x^2 - 3)^2} \\ &= \frac{4x^4 - 9x^2 + 8x}{(4x^2 - 3)^2} \end{aligned}$$

Method 1

Example 4

Find dy/dx if $4x^2y - 3y = x^3 - 1$.

 **Implicit Differentiation** We equate the derivatives of the two sides.

$$\frac{d}{dx}(4x^2y - 3y) = \frac{d}{dx}(x^3 - 1)$$

We obtain, after using the Product Rule on the first term,

$$4x^2 \cdot \frac{dy}{dx} + y \cdot 8x - 3 \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}(4x^2 - 3) = 3x^2 - 8xy$$

$$\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$

Method 2

Example 4

Find dy/dx if $4x^2y - 3y = x^3 - 1$.

Method 1

$$\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$



Method 2

$$\frac{dy}{dx} = \frac{4x^4 - 9x^2 + 8x}{(4x^2 - 3)^2}$$

$$\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$

$$= \frac{3x^2 - 8x \frac{x^3 - 1}{4x^2 - 3}}{4x^2 - 3}$$

$$= \frac{12x^4 - 9x^2 - 8x^4 + 8x}{(4x^2 - 3)^2}$$

$$= \frac{4x^4 - 9x^2 + 8x}{(4x^2 - 3)^2}$$

Summary of Implicit Differentiation

Derivation Rule of Implicit Function

Method: Derivation about x on both sides of the equation.

The function of y is a composite function of x .

Use the composite chain rule for y .

Questions and Answers



Find the equation of the tangent line to the curve

$$y^3 - xy^2 + \cos xy = 2 \text{ at the point } (0, 1).$$

We differentiate both sides, so we have,

$$\text{✎ } 3y^2 y' - x(2yy') - y^2 - (\sin xy)(xy' + y) = 0$$

$$\Rightarrow y' = \frac{y^2 + y \sin xy}{3y^2 - 2xy - x \sin xy}$$

$$\Rightarrow y'|_{(0,1)} = \frac{1}{3} \text{ so } y = \frac{1}{3}x + 1.$$

Questions and Answers



$x^4 - xy + y^4 = 1$, find the value of y'' at the point $(0,1)$

Method
1

$$4x^3 - y - xy' + 4y^3 \cdot y' = 0 \Rightarrow y' \Big|_{(0,1)} = \frac{1}{4}$$

Again derivation about x , so we have

$$12x^2 - y' - y' - xy'' + 12y^2 \cdot y' \cdot y' + 4y^3 \cdot y'' = 0$$

$$y'' \Big|_{(0,1)} = -\frac{1}{16}$$

Questions and Answers



$x^4 - xy + y^4 = 1$, find the value of y'' at the point $(0,1)$

Method
2

$$4x^3 - y - xy' + 4y^3 y' = 0$$

$$\Rightarrow y' = \frac{y - 4x^3}{4y^3 - x} \Rightarrow y' \Big|_{(0,1)} = \frac{1}{4}$$

so we have, $y' = \frac{y - 4x^3}{4y^3 - x}$

$$y'' = \frac{(y' - 12x^2)(4y^3 - x) - (y - 4x^3)(12y^2 \cdot y' - 1)}{(4y^3 - x)^2}$$

$$\Rightarrow y'' \Big|_{(0,1)} = -\frac{1}{16}$$

Questions and Answers



Find the implicit equation of $xy - e^x + e^y = 0$, and find $y', y' \Big|_{x=0}$



Let $y = y(x)$ replace y in the following equation,

$$\text{then, } xy - e^x + e^y = 0$$

derivation about x , so we have,

$$(xy)'_x - (e^x)'_x + (e^y)'_x = (0)'$$

$$y + xy' - e^x + e^y \cdot y' = 0 \implies y' \Big|_{\substack{x=0 \\ y=0}} = \frac{e^x - y}{e^y + x} \Big|_{\substack{x=0 \\ y=0}} = 1.$$

Implicit Differentiation

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